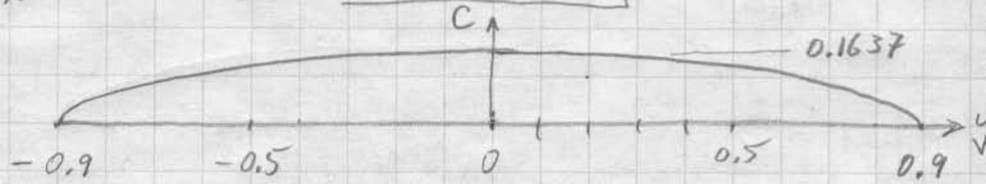
UE Fluids Problem 8 Solution S'08

1a) $L = \frac{\pi}{4}\rho V_\infty \Gamma_0 b = W$ in level flight

$\rightarrow \Gamma_0 = \frac{4}{\pi}\frac{W}{\rho V_\infty b} = \frac{4}{\pi}\frac{4N}{1.2\,kg/m^3 \cdot 6\,m/s \cdot 1.8\,m} = 0.393\ m^2/s$

$$\boxed{c(y) = \frac{2\Gamma_0}{V_\infty c_\ell}\sqrt{1-\left(\frac{2y}{b}\right)^2} = \frac{2\cdot 0.393}{6\,m/s \cdot 0.8}\sqrt{1-\left(\frac{2y}{b}\right)^2} = 0.1637\sqrt{1-\left(\frac{2y}{b}\right)^2}}$$



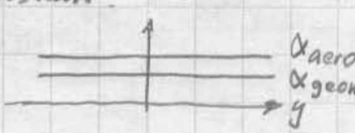
or

1b) $\alpha_i = \frac{\Gamma_0}{2bV_\infty} = \frac{0.393\,m^2/s}{2\cdot 1.8\,m \cdot 6\,m/s} = 0.0182\ rad = 1.0425°$

$$\boxed{\alpha_{aero} + \cancel{\alpha}^{0} = \frac{c_\ell}{2\pi} + \alpha_i = \frac{0.8}{2\pi} + 0.0182\ rad = 0.1455\ rad = 8.338°}\quad \text{constant}$$

1c) $\alpha_{geom} = \alpha_{aero} + \alpha_{L=0}$, for same airfoil, $\alpha_{L=0}(y) =$ constant

$\therefore$ α_{geom} is also spanwise constant, shifted by $\alpha_{L=0}$ (<0)



1d) $C_{D_i} = \alpha_i C_L = \alpha_i c_\ell$ (for elliptic loading case)

$$\boxed{C_{D_i} = 0.0182 \cdot 0.8 = 0.0146}\ ,\ \boxed{\frac{L}{D} = \frac{C_L}{C_D} = \frac{c_\ell}{c_{d_p} + C_{D_i}} = \frac{0.8}{0.015 + 0.0146} = 27.06}$$

2a) $\Gamma_0 = \frac{4}{\pi}\frac{W}{\rho V_\infty b} = \frac{4}{\pi}\frac{4N}{1.2\,kg/m^3 \cdot 12\,m/s \cdot 1.8\,m} = 0.1965\ m^2/s$

2b) $\alpha_i = \frac{\Gamma_0}{2bV_\infty} = \frac{0.1965}{2\cdot 1.8\,m \cdot 12\,m/s} = 0.00455\ rad = 0.261°$

α_{aero} is same as in 1b) (same wing). $\alpha_{aero} = 0.1455$ rad

$c_\ell = \frac{2\Gamma_0}{V_\infty c_0} = 0.200$ (4x lower than at low speed, since V_∞ is 2x higher)

$$\boxed{\alpha = \frac{c_\ell}{2\pi} - \alpha_{aero} + \alpha_i = \frac{0.2}{2\pi} - 0.1455 + 0.00455 = -0.109\ rad = -6.25°}$$